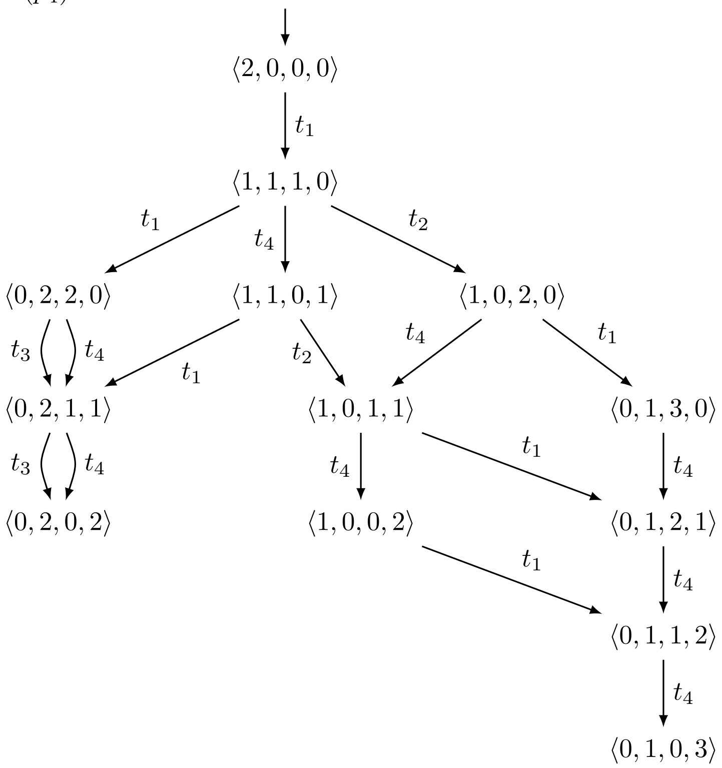
Networks and Processes

– Sample Solutions –

D. Nowotka 14.02.2008

1. (a) reachability graph where $\langle r, s, t, u\rangle$ represents a marking M with $M(p_1) = r$ and $M(p_2) = s$ and $M(p_3) = t$ and $M(p_4) = u$

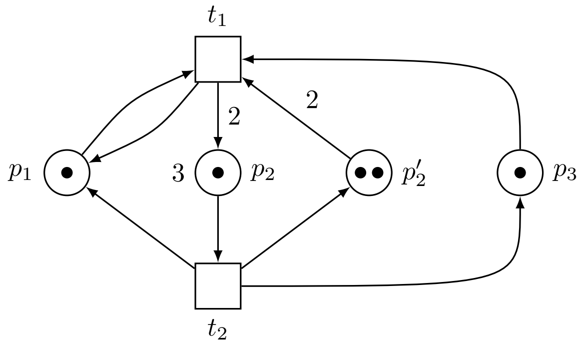


(b) pairs of independent transitions: $\{t_1, t_4\}$, $\{t_1, t_3\}$, and $\{t_2, t_4\}$

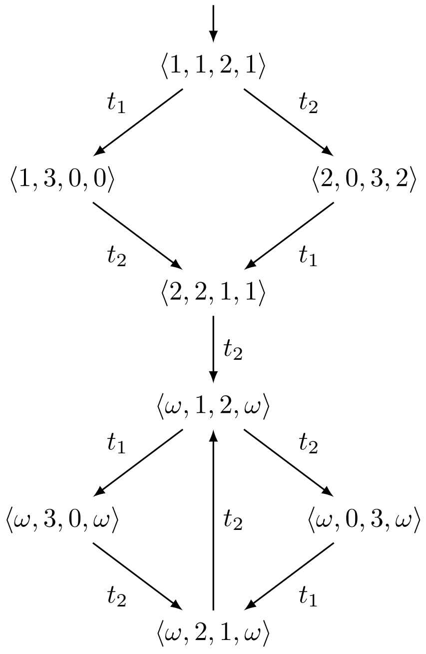
(c) yes, consider $\{t_3, t_4\}$ in $\langle 0, 2, 2, 0\rangle$

---

2. (a) net without capacities



(b) coverability graph where $\langle r, s, t, u\rangle$ represents a marking M with $M(p_1) = r$ and $M(p_2) = s$ and $M(p_2') = t$ and $M(p_3) = u$



---

3. (a) t_1 and t_3 are in conflict.

(b) $C = \begin{pmatrix} t_1 & t_2 & t_3 & t_4 & t_5 \\ 0 & 1 & -1 & 0 & 0 \\ -1 & 0 & -1 & 1 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 1 \end{pmatrix} \begin{matrix} \\ p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \end{matrix}$

(c) $D_0 = \left(\begin{array}{ccccc|ccccc} 0 & 1 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & -1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 0 & 0 & 0 & 0 & 1 \end{array}\right)$

$D_1 = \left(\begin{array}{ccccc|ccccc} 0 & 1 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 1 & -1 & 0 & 1 & 1 & 0 & 0 \end{array}\right)$

$D_2 = \left(\begin{array}{ccccc|ccccc} 0 & 0 & 1 & -1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 1 & -1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{array}\right)$

$D_3 = \left(\begin{array}{ccccc|ccccc} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right)$

$(1\ 0\ 0\ 1\ 0)^T$ and $(0\ 1\ 1\ 0\ 1)^T$ are P-invariants

(d) $\{p_1, p_2, p_5\}$ is a trap.

(e) From P-invariants, we know that for all reachable marking M:

$$M(p_1) + M(p_4) \quad = \quad M_0(p_1) + M_0(p_4) = 1 \tag{1}$$
$$M(p_2) + M(p_3) + M(p_5) \quad = \quad M_0(p_2) + M_0(p_3) + M_0(p_5) = 1 \tag{2}$$

Since the trap $\{p_1, p_2, p_5\}$ is initially marked, we have:

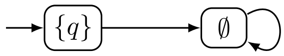
$$M(p_1) + M(p_2) + M(p_5) \quad \geq \quad 1 \tag{3}$$

$(1) + (2) - (3)$ proves that

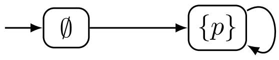
$$M(p_3) + M(p_4) \quad \leq \quad 1 \tag{4}$$

---

4. (a) i. $\mathcal{L}((\mathbf{X}\,p)\,\mathbf{U}\,q) \not\subseteq \mathcal{L}(\mathbf{X}\,(p\,\mathbf{U}\,(p\vee q)))$. The following Kripke structure satisfies $(\mathbf{X}\,p)\,\mathbf{U}\,q$ but not $\mathbf{X}\,(p\,\mathbf{U}\,(p\vee q))$.



ii. $\mathcal{L}(\mathbf{X}\,(p\,\mathbf{U}\,(p\vee q))) \not\subseteq \mathcal{L}((\mathbf{X}\,p)\,\mathbf{U}\,q)$. The following Kripke structure satisfies $\mathbf{X}\,(p\,\mathbf{U}\,(p\vee q))$ but not $(\mathbf{X}\,p)\,\mathbf{U}\,q$.



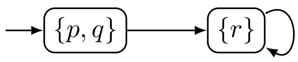
(b) $\mathcal{L}(\mathbf{G}\,(p \rightarrow \mathbf{X}\,\neg q) \rightarrow \neg\mathbf{G}\,r) = \mathcal{L}(\mathbf{F}\,(r \rightarrow (p \wedge \mathbf{X}\,q)))$. This is shown by equivalent transformations of the formulas using the equivalences given in the lecture.

$$
\begin{aligned}
\mathbf{G}\,(p \rightarrow \mathbf{X}\,\neg q) \rightarrow \neg\mathbf{G}\,r &\equiv \neg\mathbf{G}\,(\neg p \vee \mathbf{X}\,\neg q) \vee \neg\mathbf{G}\,r && \text{(definition of } \rightarrow\text{)}\\
&\equiv \mathbf{F}\,\neg(\neg p \vee \mathbf{X}\,\neg q) \vee \mathbf{F}\,\neg r && (\neg\mathbf{G}\,\varphi \equiv \mathbf{F}\,\neg\varphi)\\
&\equiv \mathbf{F}\,(\neg(\neg p \vee \mathbf{X}\,\neg q) \vee \neg r) && (\mathbf{F}\,\varphi \vee \mathbf{F}\,\varphi' \equiv \mathbf{F}\,(\varphi \vee \varphi'))\\
&\equiv \mathbf{F}\,((p \wedge \mathbf{X}\,q) \vee \neg r) && \text{(deMorgan and } \mathbf{X}\,\neg\varphi \equiv \neg\mathbf{X}\,\varphi\text{)}\\
&\equiv \mathbf{F}\,(r \rightarrow (p \wedge \mathbf{X}\,q)) && \text{(commutativity of } \vee \text{ and definition of } \rightarrow\text{)}
\end{aligned}
$$

(c) i. $\mathcal{L}(p\,\mathbf{U}\,(q \wedge r)) \subseteq \mathcal{L}((p\,\mathbf{U}\,q) \wedge (p\,\mathbf{U}\,r))$. $AP = \{p, q, r\}$.

$$
\begin{aligned}
\mathcal{L}(p\,\mathbf{U}\,(q \wedge r)) &= \{\sigma \in (2^{AP})^\omega \mid \exists i \geq 0 : q \in \sigma(i) \text{ and } \forall k < i : p \in \sigma(k) \text{ and } q \in \sigma(k)\}\\
&\subseteq \{\sigma \in (2^{AP})^\omega \mid \exists i \geq 0 : q \in \sigma(i), \forall k < i : p \in \sigma(k) \text{ and } \exists i \geq 0 : q \in \sigma(i), \forall k < i : q \in \sigma(k)\}\\
&= \mathcal{L}((p\,\mathbf{U}\,q) \wedge (p\,\mathbf{U}\,r)).
\end{aligned}
$$

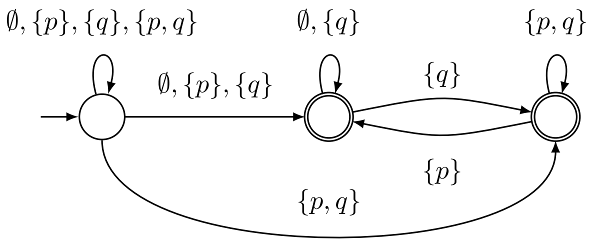
ii. $\mathcal{L}((p\,\mathbf{U}\,q) \wedge (p\,\mathbf{U}\,r)) \not\subseteq \mathcal{L}(p\,\mathbf{U}\,(q \wedge r))$. The following Kripke structure satisfies $(p\,\mathbf{U}\,q) \wedge (p\,\mathbf{U}\,r)$ but not $p\,\mathbf{U}\,(q \wedge r)$.



(d) $\mathcal{L}(\mathbf{F}\,(p\,\mathbf{U}\,q)) = \mathcal{L}(\mathbf{F}\,q)$.

$$
\begin{aligned}
\mathcal{L}(\mathbf{F}\,(p\,\mathbf{U}\,q)) &= \{\sigma \in (2^{AP})^\omega \mid \exists i \geq 0 : \exists j \geq 0 : q \in \sigma(i+j) \text{ and } \forall k : i \leq k < i+j, p \in \sigma(k)\}\\
&= \{\sigma \in (2^{AP})^\omega \mid \exists i \geq 0 : \exists j \geq 0 : q \in \sigma(i+j)\} && \text{(choose } j = 0 \text{ as witness)}\\
&= \{\sigma \in (2^{AP})^\omega \mid \exists i' \geq 0 : q \in \sigma(i')\}\\
&= \mathcal{L}(\mathbf{F}\,q).
\end{aligned}
$$

5.



---

6. (a) The pairs $\{a, c\}$, $\{b, d\}$, $\{b, e\}$, $\{d, e\}$, $\{a, a\}$, $\{b, b\}$, $\{c, c\}$, $\{d, d\}$, and $\{e, e\}$ depend on each other.

(b) i. C0 is violated.

ii. C1 and C2 are violated.

iii. C1 is violated.

iv. No condition is violated.

---

7. (a) We are given a P/T net N and an LTL formula ϕ. Assume that N is 1-safe and the set of places of N defines a set of atomic propositions where a proposition is true in a marking iff the corresponding place contains a token. (Note that other assumptions are possible. However, the P/T net has to be k-safe since only finite structures are considered for model-checking in this course.)

   i. Construct a Büchi automaton $\mathcal{B}_N$ from the reachability graph of N.
   ii. Construct a Büchi automaton $\mathcal{B}_{\neg\phi}$ from the negation of ϕ.
   iii. Construct a Büchi automaton $\mathcal{B}_\times$ with $\mathcal{L}(\mathcal{B}_\times) = \mathcal{L}(\mathcal{B}_N) \cap \mathcal{L}(\mathcal{B}_{\neg\phi})$ by intersecting $\mathcal{B}_N$ and $\mathcal{B}_{\neg\phi}$.
   iv. Check the emptiness of $\mathcal{L}(\mathcal{B}_\times)$ by searching for a reachable, non-trivial SCC in the transition graph of $\mathcal{B}_\times$ that contains an accepting state.
   v. N satisfies ϕ iff $\mathcal{L}(\mathcal{B}_\times) = \emptyset$.

(b) A Büchi automaton $\mathcal{B}$ accepts a sequence if an accepting state s can be reached from an initial state s_0 and s can be reached from s by reading at least one symbol, in other words $s_0 \rightarrow^* s \rightarrow^+ s$ where $\rightarrow$ is the transition relation of $\mathcal{B}$. The emptiness check searches for suitable s_0 and s in $\mathcal{B}$.

   i. Identify all non-tivial strongly connected components in the transition graph of $\mathcal{B}$.
   ii. Consider only those SCCs that are reachable from an initial state.
   iii. Does any of those reachable SCCs contain an accepting state? If yes, then $\mathcal{L}(\mathcal{B}) \neq \emptyset$. Otherwise $\mathcal{L}(\mathcal{B}) = \emptyset$.

Note that an efficient algorithm for the emptiness check might interleave the steps above.
