D. Nowotka | Winter Term 2008/09

Networks and Processes

Exercise Sheet 5

Discussion on – 15.1.2009, 15h45 (submissions of solutions are highly recommended, either before or after the exercise session)

1. In this problem we consider a variant of Büchi automata, called transition Büchi automata (TBA). Let $\mathcal{A} = \langle \Sigma, S, S^0, \Delta, T \rangle$ be a TBA, where

   - Σ is a finite alphabet,
   - S is a finite set of states,
   - $S^0 \subseteq S$ are the initial states,
   - $\Delta \subseteq S \times \Sigma \times S$ is the transition relation, and
   - $T \subseteq \Delta$ are the acceptance transitions.

   Let $\sigma = a_1 a_2 a_3 \ldots \in \Sigma^\omega$ be an infinite word. A run of $\mathcal{A}$ on σ is an infinite sequence $\rho = s_0 a_1 s_1 a_2 s_2 \ldots$, such that $s_0 \in S^0$ and $(s_{i-1}, a_i, s_i) \in \Delta$ for all $i \geq 1$. Let $\inf(\rho)$ be the set of transitions (s, a, s') that appear infinitely often in ρ. A word σ is accepted by $\mathcal{A}$ if there is some run ρ of $\mathcal{A}$ on σ with $\inf(\rho) \cap T \neq \emptyset$.

   Translate the TBA $\mathcal{A}$ into an equivalent Büchi automaton $\mathcal{B}$, where its acceptance is defined by accepting states.

2. Given LTL formula $\phi \equiv \mathbf{F}\, p$. Answer the following questions.

   a) Convert ϕ into an equivalent negated normal form.
   b) Give the set of subformulas $Sub(\phi)$.
   c) Give the consistent subsets $CS(\phi)$ of $Sub(\phi)$.
   d) Give a pre-Hintikka sequence for formula ϕ that is not a Hintikka sequence.
   e) Give a Hintikka sequence for formula ϕ.
   f) Construct a Büchi automaton $\mathcal{B}$ such that $\mathcal{L}(\phi) = \mathcal{L}(\mathcal{B})$ by using the method presented in the lecture.

3. For each LTL formula ϕ (over AP) below, construct a Büchi automaton $\mathcal{B}$ (over the alphabet 2^{AP}) such that $\mathcal{L}(\phi) = \mathcal{L}(\mathcal{B})$. You may do the construction in an ad hoc manner or use the method presented in the lecture.

   a) $\mathbf{F}\,(p \wedge \mathbf{X}\, p)$, where $AP = \{p\}$.
   b) $\mathbf{G}\,((p \wedge \mathbf{X}\, q) \rightarrow \mathbf{F}\,(p \wedge q))$, where $AP = \{p, q\}$.
   c) $(\mathbf{F}\, p)\ \mathbf{U}\ q$, where $AP = \{p, q\}$.