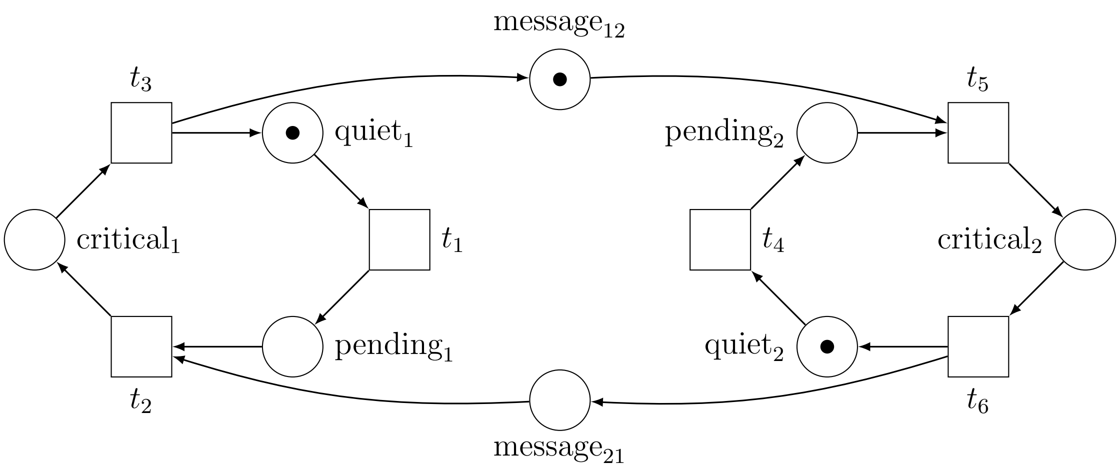
D. Nowotka Winter Term 2008/09

Networks and Processes

Exercise Sheet 6

Discussion on – 22.1.2008, 15h45 (submissions of solutions are highly recommended, either before or after the exercise session)

1. Consider the net N below which addesses the alternating mutual exclusion problem of two processes.



Follow step-by-step the outline given below to model check the following formulas on the net N.

a) $f_a := \mathbf{F}\,\mathbf{G}\,\neg\text{critical}_1$, where $AP_a = \{\text{critical}_1\}$

b) $f_b := \mathbf{G}\,(\text{pending}_1 \rightarrow \mathbf{F}\,\text{critical}_1)$, where $AP_b = \{\text{pending}_1, \text{critical}_1\}$

Model checking an LTL formula f over atomic propositions AP in a P/T net N can be performed in the following steps:

(i) Generate the reachability graph $G_N = (V, E, M_0)$ of the net N.

(ii) Generate a Büchi automaton $\mathcal{B}_{\neg f}$ for the negation of the formula f.

(iii) Generate the intersection Büchi automaton $\mathcal{B}$ for $\mathcal{B}_{\mathcal{K}}$ and $\mathcal{B}_{\neg f}$, where $\mathcal{B}_{\mathcal{K}}$ is the Büchi automaton over the alphabet 2^{AP} obtained from the reachability graph G_N. Note that a marking M is labelled by an atomic proposition $p \in AP$ if and only if the place p contains at least one token, i.e., $M(p) \geq 1$. (The intersection automaton $\mathcal{B}$ accepts all the words in $\mathcal{L}(\mathcal{B}_{\mathcal{K}}) \cap \mathcal{L}(\mathcal{B}_{\neg f})$.)

(iv) Check whether $\mathcal{L}(\mathcal{B}) \stackrel{?}{=} \emptyset$:

please turn overleaf ⟲

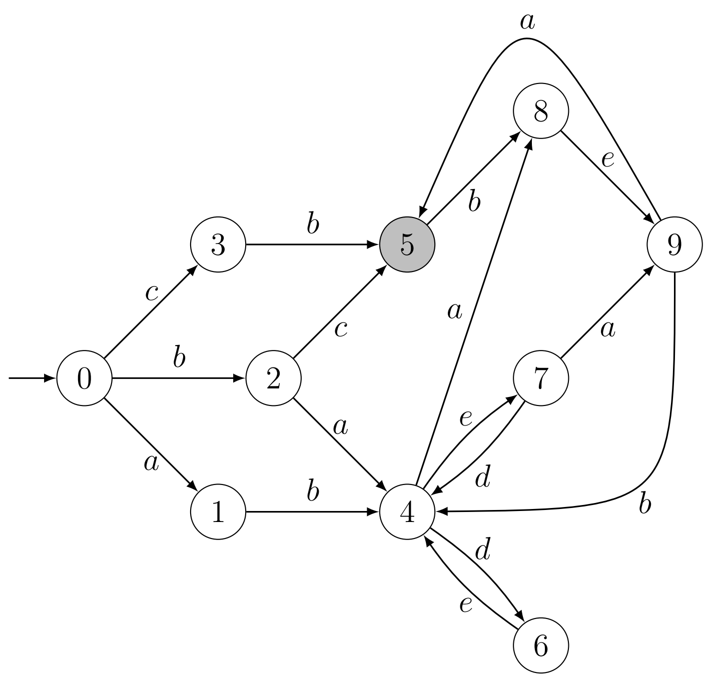
Case a) $\mathcal{L}(\mathcal{B}) = \emptyset$: The formula holds, i.e. $\mathcal{K} \models f$.

Case b) $\mathcal{L}(\mathcal{B}) \neq \emptyset$: The formula does not hold, i.e. $\mathcal{K} \not\models f$. In this case, generate a counterexample run r of the net N that violates the property given by the LTL formula f. The counterexample is obtained by finding an infinite word accepted by the intersection Büchi automaton $\mathcal{B}$ and projecting it on the first component (i.e. the automaton $\mathcal{B}_{\mathcal{K}}$ associated to the net N) to get an infinite sequence of consecutive markings of N.

First do step (i), and then the steps (ii)–(iv) separately for f_a and AP_a, and separately for f_b and AP_b. Write down all the intermediary results.

Hint: For the given formulas, it is easy to come up with the automaton $\mathcal{B}_{\neg f}$ directly! (Simulating the LTL to Büchi automaton translation procedure from the lecture notes is likely to consume a *lot* of paper.)

2. Consider the Kripke structure $\mathcal{K}$ below with its transitions labeled by the actions $A = \{a, b, c, d, e\}$ and with only one atomic proposition $AP = \{p\}$ with $\nu(p) = \{5\}$.



a) Indicate all pairs of actions that depend on each other.

b) Indicate which actions from A are visible and which are not.

c) Compute a reduction function *red* that satisfies the conditions C0–C3 (Definition 3.17) on the slide. Whenever possible, choose *red*(s) such that it is a proper subset of *en*(s), for each state s. Use *red* to obtain a reduced Kripke structure *red*$(\mathcal{K})$ that is stuttering equivalent to the original Kripke structure $\mathcal{K}$.