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Networks and Processes

Exercise Sheet 7

Discussion on – 22.1.2009, 15h45 (submissions of solutions are highly recommended, either before or after the exercise session)

1. Which of following pairs of formulas are equivalent when ϕ_1, ϕ_2 and ϕ_3 are arbitrary CTL formulas? Provide a proof if equivalent, or a counterexample otherwise.

   a) $\mathbf{AF}\,\mathbf{AF}\,\phi_1$, $\mathbf{AF}\,\phi_1$

   b) $\mathbf{EF}\,\phi_1 \wedge \mathbf{EG}\,\phi_2$, $\mathbf{EF}\,(\phi_1 \wedge \mathbf{EG}\,\phi_2)$

   c) $(\phi_1\ \mathbf{EU}\ \phi_2) \wedge (\phi_2\ \mathbf{EU}\ \phi_3)$, $\phi_1\ \mathbf{EU}\ \phi_3$

   d) $\phi_1\ \mathbf{AU}\ \phi_2$, $\neg(\neg\phi_2\ \mathbf{EU}\ (\neg\phi_1 \wedge \neg\phi_2) \vee \mathbf{EG}\,\neg\phi_2)$

2. Given an arbitrary CTL formula ϕ, prove that the following equivalences hold:

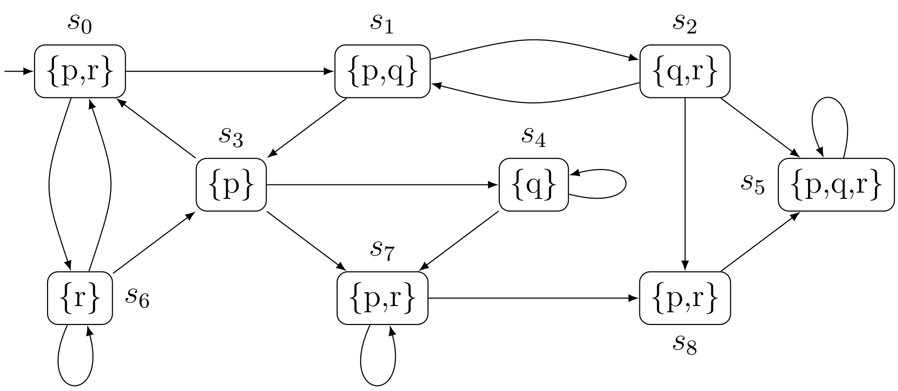
$$\mathbf{EF}\,\mathbf{AG}\,\mathbf{EF}\,\mathbf{AG}\,\phi \equiv \mathbf{EF}\,\mathbf{AG}\,\phi$$
$$\mathbf{AG}\,\mathbf{EF}\,\mathbf{AG}\,\mathbf{EF}\,\phi \equiv \mathbf{AG}\,\mathbf{EF}\,\phi$$

   Hint: Prove the following implications first.

$$\mathbf{EF}\,\mathbf{AG}\,\phi \Rightarrow \mathbf{EF}\,\phi \qquad (1)$$
$$\mathbf{AG}\,\phi \Rightarrow \mathbf{AG}\,\mathbf{EF}\,\mathbf{AG}\,\phi \qquad (2)$$

   Note that for two arbitrary CTL formulas ϕ and ψ, we say that $\phi \Rightarrow \psi$ iff for all Kripke structures $\mathcal{K}$ and states s, $s \in [\![\phi]\!]_{\mathcal{K}}$ implies $s \in [\![\psi]\!]_{\mathcal{K}}$.

3. Consider the Kripke structure $\mathcal{K}$, where $AP = \{p, q, r\}$:



   Determine the sets of states in which the following CTL formulas hold. I.e., compute $[\![\phi_i]\!]$, for every $i \in \{1, \ldots, 5\}$.

please turn overleaf ⥀

a) $\phi_1 = p \, \mathbf{EU} \, q$

b) $\phi_2 = r \wedge (p \, \mathbf{AU} \, q)$

c) $\phi_3 = \mathbf{EX} \, (r \wedge (p \, \mathbf{AU} \, q))$

d) $\phi_4 = \mathbf{AF} \, q$

e) $\phi_5 = \mathbf{EG_f} \, (q \vee r)$ with the fairness constraint $\{s_0, s_7, s_8\}$.

4. Give BDDs for the formula $(x_1 \leftrightarrow x_2) \rightarrow (x_3 \wedge x_4)$ by using the following orderings

a) $x_1 < x_2 < x_3 < x_4$

b) $x_1 < x_3 < x_4 < x_2$