

Networks and Processes

Sample Solutions to Exercise 4

Discussion on – 18.12.2008, 15h45 (submissions of solutions are highly recommended, either before or after the exercise session)

1. a) No b) Yes c) Yes
 d) No e) Yes f) Yes
 g) Yes h) No i) No

2. a) $\mathbf{G}(send \rightarrow \mathbf{X}(\neg send \mathbf{W} ack))$
 b) $\mathbf{G} \mathbf{F} rec \rightarrow \mathbf{G} \mathbf{F} send$
 c) $\mathbf{G}(req \rightarrow \mathbf{F} \neg req)$
 d) $\mathbf{F}(\mathbf{G} cs_1 \rightarrow \mathbf{G} \neg req_2)$
 e) $\mathbf{F}(\mathbf{G} b \vee \mathbf{G} \neg b)$
 f) $\mathbf{G}(req \rightarrow (\neg cs \mathbf{W} b))$

3. a) Not equivalent. The word $\{\}\{p\}\{p\}\{p\} \dots \in \mathcal{L}(\psi)$, but $\notin \mathcal{L}(\phi)$.
 b) Not equivalent. The word $\{p\}\{\}\{\}\{\}\dots \in \mathcal{L}(\phi)$, but $\notin \mathcal{L}(\psi)$.
 c) Not equivalent. The word $\{\}\{p, q\}\{p\}\{p\}\{p\} \dots \in \mathcal{L}(\psi)$, but $\notin \mathcal{L}(\phi)$.
 d) Not equivalent. The word $\{p, q\}\{p\}\{p\}\{p\} \dots \in \mathcal{L}(\phi)$, but $\notin \mathcal{L}(\psi)$.
 e) Not equivalent. The word $\{q\}\{r\}\{\}\{\}\{\}\dots \in \mathcal{L}(\phi)$, but $\notin \mathcal{L}(\psi)$.
 f) Equivalent.
 “ \Rightarrow ” Let $\sigma \in \mathcal{L}(\phi)$. If $q \in \sigma(0)$, then obviously $\sigma \in \mathcal{L}(\psi)$. Let $i \geq 1$ be the first position where q holds. Then, $p \mathbf{U} q$ holds at all positions between 0 and $j - 1$. Since q does not hold between 0 and $j - 1$, p must hold at all positions between 0 and $j - 1$. Therefore, $\sigma \in \mathcal{L}(\psi)$.
 “ \Leftarrow ” Let $\sigma \in \mathcal{L}(\psi)$. If $q \in \sigma(0)$, then obviously $\sigma \in \mathcal{L}(\phi)$. Let $i \geq 1$ be the first position where q holds. Then, p holds at all positions between 0 and $j - 1$. Since q does not hold between 0 and $j - 1$, $p \mathbf{U} q$ also holds at all positions between 0 and $j - 1$. Therefore, $\sigma \in \mathcal{L}(\phi)$.