D. Nowotka | Winter Term 2008/09

Networks and Processes

Sample Solutions to Exercise 7

Discussion on – 22.1.2009, 15h45 (submissions of solutions are highly recommended, either before or after the exercise session)

1. a) Equivalent.
   b) Not equivalent.
   c) Not equivalent.
   d) Equivalent.

2. (i) Proof of (1).
   If a Kripke structure $\mathcal{K} \models \mathbf{EF}\,\mathbf{AG}\,\phi$,
   $\Rightarrow$ there exists a run ρ starting at s^0, $\exists i \geq 0$ s.t. $\rho(i) \in [\![\mathbf{AG}\,\phi]\!]$.
   $\Rightarrow$ $\forall j \geq i : \rho(j) \in [\![\phi]\!]$.
   $\Rightarrow$ $\rho(i) \in [\![\phi]\!]$.
   $\Rightarrow$ $\mathcal{K} \models \mathbf{EF}\,\phi$.

   (ii) Proof of (2).
   If a Kripke structure $\mathcal{K} \models \mathbf{AG}\,\phi$,
   $\Rightarrow$ for all runs ρ starting at s^0, $\forall i \geq 0$ s.t. $\rho(i) \in [\![\phi]\!]$.
   $\Rightarrow$ $\rho(i) \in [\![\mathbf{AG}\,\phi]\!]$.
   $\Rightarrow$ $\rho(i) \in [\![\mathbf{EF}\,\mathbf{AG}\,\phi]\!]$.
   $\Rightarrow$ $\mathcal{K} \models \mathbf{AG}\,\mathbf{EF}\,\mathbf{AG}\,\phi$.

   (iii) $\mathbf{EF}\,\mathbf{AG}\,\mathbf{EF}\,\mathbf{AG}\,\phi \overset{(1)}{\Rightarrow} \mathbf{EF}\,\mathbf{EF}\,\mathbf{AG}\,\phi \equiv \mathbf{EF}\,\mathbf{AG}\,\phi$.

   (iv) $\mathbf{AG}\,\phi \overset{(2)}{\Rightarrow} \mathbf{AG}\,\mathbf{EF}\,\mathbf{AG}\,\phi$, so $\mathbf{EF}\,\mathbf{AG}\,\phi \Rightarrow \mathbf{EF}\,\mathbf{AG}\,\mathbf{EF}\,\mathbf{AG}\,\phi$.

   (v) From (iii) and (iv), $\mathbf{EF}\,\mathbf{AG}\,\mathbf{EF}\,\mathbf{AG}\,\phi \equiv \mathbf{EF}\,\mathbf{AG}\,\phi$.

   (vi) From (v), since ϕ is an arbitrary CTL formula, we can negate both sides of the equivalence and obtain $\mathbf{AG}\,\mathbf{EF}\,\mathbf{AG}\,\mathbf{EF}\,\phi \equiv \mathbf{AG}\,\mathbf{EF}\,\phi$.

3. a) $[\![p\ \mathbf{EU}\ q]\!] = S \setminus \{s_6\}$.
   b) $[\![r \wedge (p\ \mathbf{AU}\ q)]\!] = \{s_2, s_5, s_8\}$.
   c) $[\![\mathbf{EX}\,(r \wedge (p\ \mathbf{AU}\ q))]\!] = \{s_1, s_2, s_5, s_7, s_8\}$.
   d) $[\![\mathbf{AF}\,q]\!] = \{s_1, s_2, s_4, s_5, s_8\}$.
   e) $[\![\mathbf{EG}_\mathbf{f}\,(q \vee r)]\!] = \{s_0, s_4, s_6, s_7\}$. Refer to the slide page 415.
      (i) $\mathcal{K}_{q \vee r}$ contains every state of $\mathcal{K}$ except s_3.
      (ii) The non-trivial SCCs are $\{s_0, s_6\}, \{s_1, s_2\}, \{s_4\}, \{s_5\}, \{s_7\}$.

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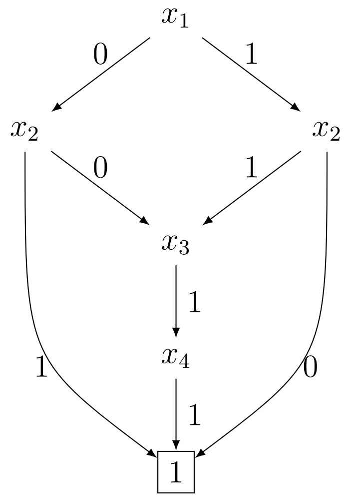
(iii) The non-trivial SCCs that contain a state from the fairness constrain are $\{s_0, s_6\}$, $\{s_4\}$, and $\{s_7\}$.

(iv) $\mathbf{EG_f}\,(q \vee r)$ contains states in $\mathcal{K}_{q \vee r}$ that can reach $\{s_0, s_4, s_6, s_7\}$.

4. a)



b)