Networks and Processes

– Sample Solutions –

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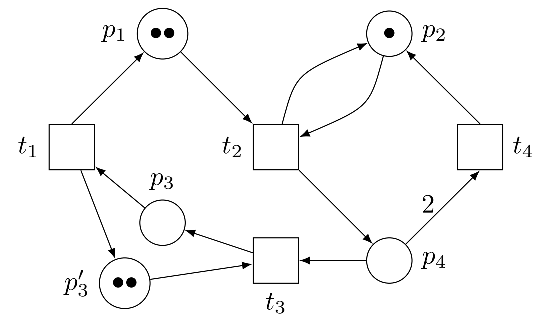
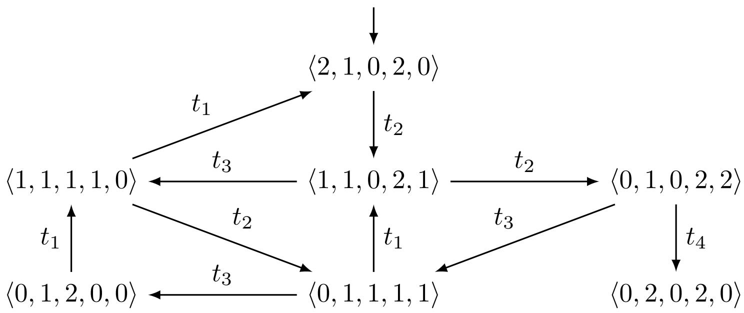
1. (a) Consider N'



(b) $\{t_2, t_3\}$

(c) reachability graph where $\langle r, s, t, u, v\rangle$ represents a marking M with $M(p_1) = r$ and $M(p_2) = s$ and $M(p_3) = t$ and $M(p'_3) = u$ and $M(p_4) = v$



(d) N' is not deadlock-free. It may reach the deadlock $\langle 0, 2, 0, 2, 0\rangle$.

(e) (t_1, t_3) and (t_2, t_3) form pairs of independent transitions.

(f) $$C = \begin{array}{c} \\ \\ \\ \\ \\ \end{array}\left(\begin{array}{rrrr} t_1 & t_2 & t_3 & t_4 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & -2 \end{array}\right)\begin{array}{l} \\ p_1 \\ p_2 \\ p_3 \\ p'_3 \\ p_4 \end{array}$$

(g) $$D_0 = \left(\begin{array}{rrrr|rrrrr} 1 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & -1 & -2 & 0 & 0 & 0 & 0 & 1 \end{array}\right)$$

$$D_1 = \left(\begin{array}{rrrr|rrrrr} 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -2 & 0 & 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{array}\right)$$

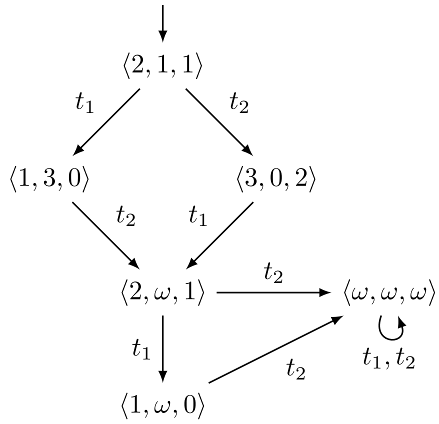
$$D_2 = \left(\begin{array}{rrrr|rrrrr} 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & -2 & 1 & 0 & 1 & 0 & 1 \end{array}\right)$$

$$D_3 = \left(\begin{array}{rrrr|rrrrr} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 & 1 & 0 & 1 \end{array}\right)$$

$(0\ 0\ 1\ 1\ 0)^T$ and $(1\ 2\ 1\ 0\ 1)^T$ are P-invariants

(h) $\{p_2\}$ and $\{p_1, p_2\}$ are traps.

2. (a) coverability graph where $\langle r, s, t\rangle$ represents a marking M with $M(p_1) = r$ and $M(p_2) = s$ and $M(p_3) = t$



(b) No, one cannot conclude in general that the network is deadlock-free (although it is in this particular case) since the coverability graph consists of coverings of markings, that is, a deadlock may be covered by a marking which is not a deadlock.

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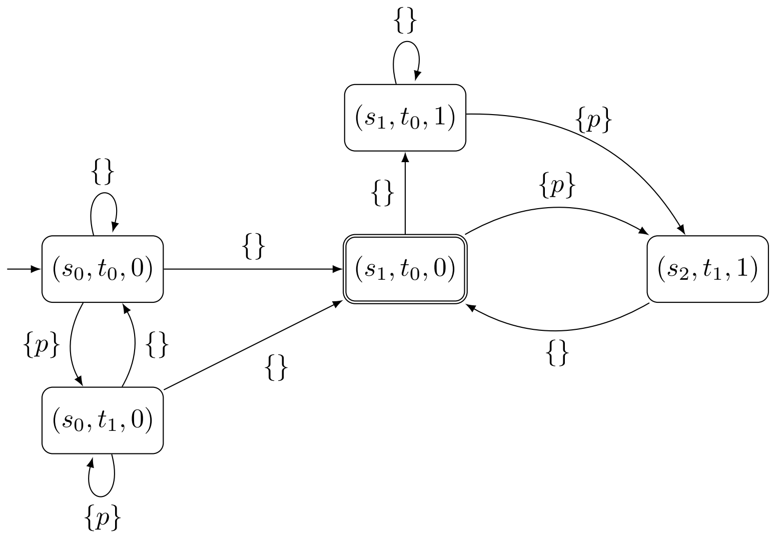
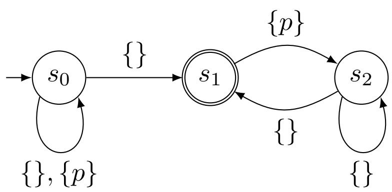
3. Büchi automata

(a) Consider



(b) Consider



(c) Consider

$$\phi = \mathbf{F}\,\mathbf{G}\,(p \to \mathbf{X}\,\neg p) \qquad \text{and} \qquad \psi = \mathbf{G}\,\mathbf{F}\,p$$

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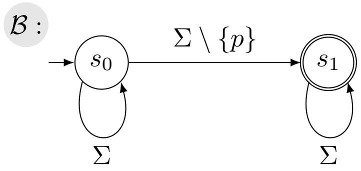
4. (a) $\mathbf{F}\,(p \to q) \equiv \mathit{true}\ \mathbf{U}\ (\neg p \vee q)$

$$\mathit{Sub}(\phi) = \{\mathit{true}, \mathit{false}, p, \neg p, q, \neg q, \neg p \vee q, \neg(\neg p \vee q), \mathit{true}\ \mathbf{U}\ (\neg p \vee q), \neg(\mathit{true}\ \mathbf{U}\ (\neg p \vee q))\}$$

(b)

$$\begin{aligned} CS(\phi) = \{&\{\mathit{true}, \neg p \vee q\}, \{\mathit{true}, p\}, \{\mathit{true}, p, q, \neg p \vee q\}, \{\mathit{true}, q, \neg p \vee q\}, \{\mathit{true}, \neg p \vee q, \mathit{true}\ \mathbf{U}\ (\neg p \vee q)\},\\ &\{\mathit{true}, p, \mathit{true}\ \mathbf{U}\ (\neg p \vee q)\}, \{\mathit{true}, p, q, \neg p \vee q, \mathit{true}\ \mathbf{U}\ (\neg p \vee q)\}, \{\mathit{true}, q, \neg p \vee q, \mathit{true}\ \mathbf{U}\ (\neg p \vee q)\}\}\end{aligned}$$

(c) Consider



where $\Sigma = \{\{\}, \{p\}, \{q\}, \{p, q\}\}$.

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5. (a) Let $S = \{s_0, s_1, \ldots s_5\}$.
   i. $S \setminus \{s_1\}$
   ii. $\{s_1, s_2, s_3\}$
   iii. S

   (b) Consider